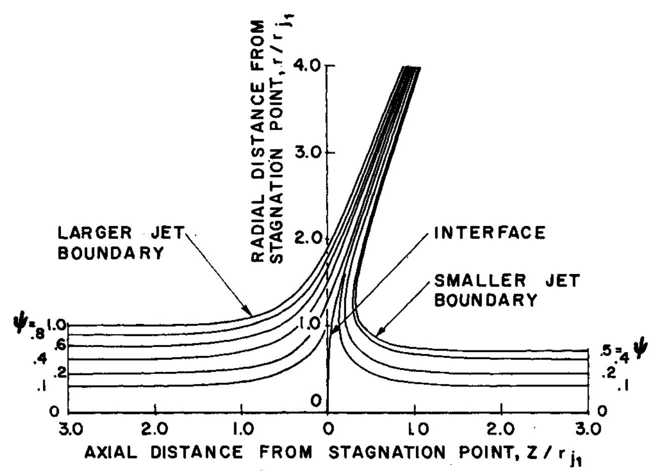

Fig. 5 Flow streamlines in unequal, opposite, circular jets.

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Elastic Behavior of Multilayered Bidirectional Composites

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SEVERAL recent papers[1–5] have addressed the problem of defining the exact (elastic) response of composite laminates under static bending. However, all of these studies have treated laminates consisting of only a few layers, while in practical applications, composite structures may consist of many layers, in some cases, 100 or more. It is therefore appropriate that we consider the response of multi-ply laminates with a view toward examining the generality of previous conclusions regarding the range of validity of the approximate theory normally used in the analysis of these bodies, namely, classical laminated plate theory (CPT).[6]

Previous investigations[1–5] have shown that the exact solution for a particular composite laminate converges to the respective CPT solution as the aspect ratio becomes very large. Furthermore, the exact solution for the stress field generally converges more rapidly than that for deflection. In particular, a good estimate for the maximum normal stress is given by CPT for aspect ratios as low as 10, while for highly anisotropic materials, the error in the CPT deflection estimate can be appreciable for aspect ratios as high as 30. The objective of the present work is to examine these trends for laminates composed of many layers. Specifically, we treat the boundary value problem discussed in Ref. 2 for square bidirectional laminates of edge dimension a and thickness h consisting of 3, 5, 7, and 9 layers under the surface loading

$$q_0 = \sigma \sin(\pi x/a) \sin(\pi y/a) \qquad (0 \leqq x \leqq a,\ 0 \leqq y \leqq a,\ -h/2 \leqq z \leqq h/2) \tag{1}$$

where σ is a constant. Each layer is a unidirectional fiber reinforced composite possessing the following engineering constants:

$$E_L = 25\times10^6\,\text{psi},\quad E_T = 10^6\,\text{psi},\quad G_{LT} = 0.5\times10^6\,\text{psi} \qquad G_{TT} = 0.2\times10^6\,\text{psi},\quad \nu_{LT} = \nu_{TT} = 0.25 \tag{2}$$

where L signifies the fiber direction, T the transverse direction, ν_{LT} is the major Poisson ratio, and the material is assumed to be square-symmetric. All laminates considered are symmetric with respect to the central plane, with fiber orientations alternating

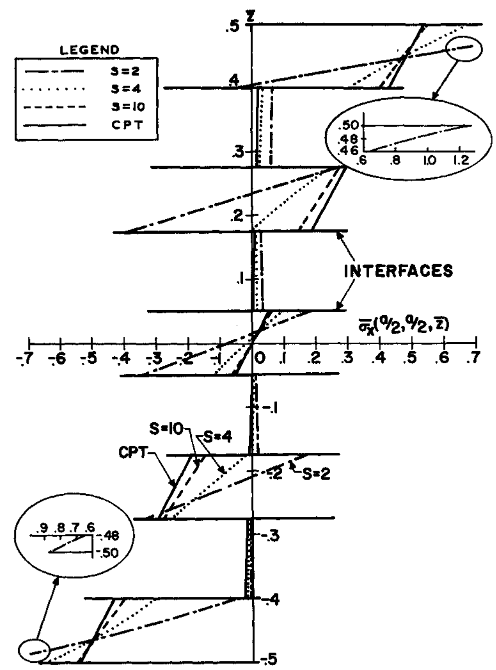

Fig. 1 Normal stress distribution.

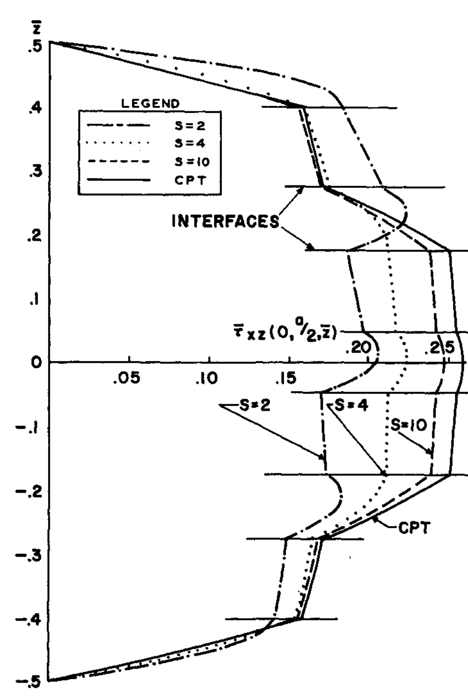

Fig. 2 Shear stress distribution.

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between 0° and 90° with respect to the x axis, and the 0° layers are at the outer surfaces of the laminate. The total thickness of the 0° and 90° layers are the same, whereas layers at the same orientation have equal thicknesses. Under these conditions, the effective laminate stiffnesses in the x and y directions are the same; also the CPT solution for deflection w and the maximum values of stress components σ_x and τ_{xy} are independent of the number of layers N for $N > 1$.

Certain aspects of the exact solutions, using the formulation developed in Ref. 2, as well as the analogous CPT results, are given in Tables 1–4 and Figs. 1–3 in terms of the following normalized quantities:

$$(\bar{\sigma}_x, \bar{\sigma}_y, \bar{\tau}_{xy}) = (1/\sigma S^2)(\sigma_x, \sigma_y, \tau_{xy}), \quad (\bar{\tau}_{xz}, \bar{\tau}_{yz}) = (1/\sigma S)(\tau_{xz}, \tau_{yz}) \tag{3}$$

$$\bar{u} = E_T u/\sigma h S^3, \quad \bar{w} = \pi^4 Q w/12 S^4 h\sigma, \quad S = a/h, \quad \bar{z} = z/h$$

where

$$Q = 4G_{LT} + [E_L + E_T(1 + 2\nu_{TT})]/(1 - \nu_{LT}\nu_{TL}) \tag{4}$$

In terms of the normalized functions (3), the CPT solution is independent of S.

The normalized stresses and deflection at the points where the CPT values are maximized are shown in Tables 1–4 for various values of S. The arguments of the functions are taken as $(x, y, \bar{z})$. In a few instances (small S) the maximum values of $\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$ do not occur at $\bar{z} = 0$ (see Fig. 2), hence in these cases there are two entries in the respective columns. The upper value gives the function at $\bar{z} = 0$, while the lower number represents the maximum value, with the corresponding $\bar{z}$ being given in parentheses.

The results of Tables 1–4 suggest that the functions presented there generally converge faster to the respective CPT values as the number of layers increases. Specifically, although CPT errs considerably in certain stress components for small N, at an aspect

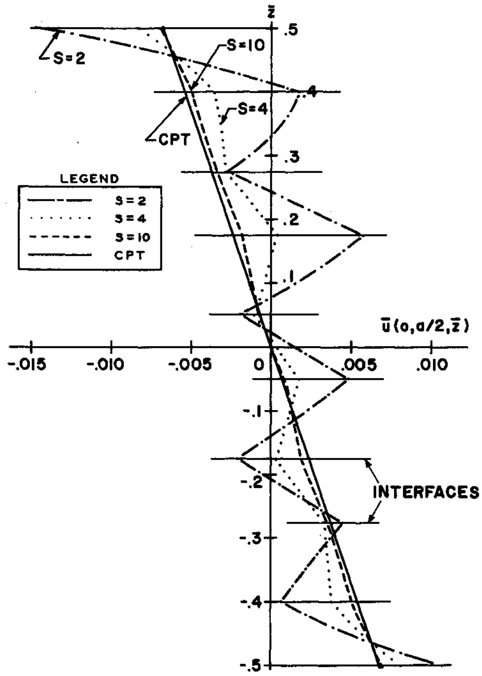

Fig. 3 In-plane displacement.

Table 1 Maximum stresses and deflection in 3-ply laminate

S	$\bar{\sigma}_x$ $(a/2, a/2, \pm 1/2)$	$\bar{\sigma}_y$ $(a/2, a/2, \pm 1/4)$	$\bar{\tau}_{xz}$ $(0, a/2, 0)$	$\bar{\tau}_{yz}$ $(a/2, 0, 0)$	$\bar{\tau}_{xy}$ $(0, 0, \pm 1/2)$	$\bar{w}$ $(a/2, a/2, 0)$
			Elasticity			
2	1.388	0.835	0.153	0.295	−0.0863	11.767
	−0.912	−0.795	0.264(0.35)	0.298(0.05)	0.0673	
4	0.720	0.663	0.219	0.292	−0.0467	4.491
	−0.684	−0.666	0.222(−0.27)		0.0458	
10	±0.559	0.401	0.301	0.196	−0.0275	1.709
		−0.403			0.0276	
20	±0.543	0.308	0.328	0.156	∓0.0230	1.189
		−0.309				
50	±0.539	±0.276	0.337	0.141	∓0.0216	1.031
100	±0.539	±0.271	0.339	0.139	∓0.0214	1.008
			CPT			
	±0.539	±0.269	0.339	0.138	∓0.0213	1

Table 2 Maximum stresses and deflection in 5-ply laminate

S	$\bar{\sigma}_x$ $(a/2, a/2, \pm 1/2)$	$\bar{\sigma}_y$ $(a/2, a/2, \pm 1/3)$	$\bar{\tau}_{xz}$ $(0, a/2, 0)$	$\bar{\tau}_{yz}$ $(a/2, 0, 0)$	$\bar{\tau}_{xy}$ $(0, 0, \pm 1/2)$	$\bar{w}$ $(a/2, a/2, 0)$
			Elasticity			
2	1.332	1.001	0.227	0.186	−0.0836	12.278
	−0.903	−0.848	0.229(0.02)	0.268(0.18)	0.0634	
4	0.685	0.633	0.238	0.229	−0.0394	4.291
	−0.651	−0.626	0.238(0.02)	0.233(−0.11)	0.0384	
10	±0.545	0.430	0.258	0.223	−0.0246	1.570
		−0.432		0.223(−0.02)	0.0247	
20	±0.539	±0.380	0.268	0.212	∓0.0222	1.145
50	±0.539	±0.363	0.271	0.206	∓0.0214	1.023
100	±0.539	±0.360	0.272	0.205	∓0.0213	1.006
			CPT			
	±0.539	±0.359	0.272	0.205	∓0.0213	1

Table 3 Maximum stresses and deflection in 7-ply laminate

S	$\bar{\sigma}_x$ $(a/2, a/2, \pm 1/2)$	$\bar{\sigma}_y$ $(a/2, a/2, \pm 3/8)$	$\bar{\tau}_{xz}$ $(0, a/2, 0)$	$\bar{\tau}_{yz}$ $(a/2, 0, 0)$	$\bar{\tau}_{xy}$ $(0, 0, \pm 1/2)$	$\bar{w}$ $(a/2, a/2, 0)$
			Elasticity			
2	1.284	1.039	0.178	0.238	−0.0775	12.342
	−0.880	−0.838	0.229(0.16)	0.239(0.02)	0.0579	
4	0.679	0.623	0.219	0.236	−0.0356	4.153
	−0.645	−0.610	0.223(0.12)		0.0347	
10	±0.548	0.457	0.255	0.219	−0.0237	1.529
		−0.458	0.255(−0.02)		0.0238	
20	±0.539	0.419	0.267	0.210	∓0.0219	1.133
		−0.420				
50	±0.539	±0.407	0.271	0.206	∓0.0214	1.021
100	±0.539	±0.405	0.272	0.205	∓0.0213	1.005
			CPT			
	±0.539	±0.404	0.272	0.205	∓0.0213	1

Table 4 Maximum stresses and deflection in 9-ply laminate

S	$\bar{\sigma}_x$ $(a/2, a/2, \pm 1/2)$	$\bar{\sigma}_y$ $(a/2, a/2, \pm 2/5)$	$\bar{\tau}_{xz}$ $(0, a/2, 0)$	$\bar{\tau}_{yz}$ $(a/2, 0, 0)$	$\bar{\tau}_{xy}$ $(0, 0, \pm 1/2)$	$\bar{w}$ $(a/2, a/2, 0)$
			Elasticity			
2	1.260	1.051	0.204	0.194	−0.0722	12.288
	−0.866	−0.824	0.224(0.23)	0.211(−0.10)	0.0534	
4	0.684	0.628	0.223	0.223	−0.0337	4.079
	−0.649	−0.612	0.223(0.01)	0.225(−0.06)	0.0328	
10	±0.551	±0.477	0.247	0.226	−0.0233	1.512
				0.226(−0.01)	0.0235	
20	±0.541	±0.444	0.255	0.221	∓0.0218	1.129
50	±0.539	±0.433	0.258	0.219	∓0.0214	1.021
100	±0.539	±0.431	0.259	0.219	∓0.0213	1.005
			CPT			
	±0.539	±0.431	0.259	0.219	∓0.0213	1

ratio $S = 10$, the maximum stresses are all predicted to within 10% accuracy by CPT in the 9-layer laminate. However, the exact deflection in this case is still 51% higher than the CPT result. Thus, as in previous studies, the stresses generally converge to the CPT values faster than the deflection.

Curves showing the thickness distribution of stresses $\bar{\sigma}_x$, $\bar{\tau}_{xz}$ and displacement $\bar{u}$ along the vertical lines, $x =$ const, $y =$ const, on which each of the functions assumes its maximum value, are given in Figs. 1–3 for the 9-ply laminate. The exact results are indicated for three different aspect ratios, $S = 2$, 4, 10 and the CPT results (independent of S) are also shown. To avoid loss of detail in the curves of Fig. 1, it is necessary to utilize the inserts representing the behavior of $\bar{\sigma}_x$ near $\bar{z} = \pm\frac{1}{2}$ for the case $S = 2$. Rapid convergence of the exact functions to CPT can be observed despite the characteristic zig-zag appearance of the deformed normal as shown in Fig. 3.

In conclusion, the preceding results suggest that a conservative estimate of the magnitude of the error reflected in the simplifying assumptions of CPT for multilayered systems can be achieved by comparison of exact and approximate solutions for laminates consisting of only several layers.

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A Laser Velocimeter for Reynolds Stress and Other Turbulence Measurements

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Nomenclature

c = speed of light
$\hat{e}_x, \hat{e}_y$ = unit vectors in the x and y directions

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